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Anonymous credentials

A critique of identity

- Identity as a proxy to check credentials
  - Username decides access in Access Control Matrix
- Sometime it leaks too much information
- Real world examples
  - Tickets allow you to use cinema / train
  - Bars require customers to be older than 18
    - But do you want the barman to know your address?

The privacy-invasive way

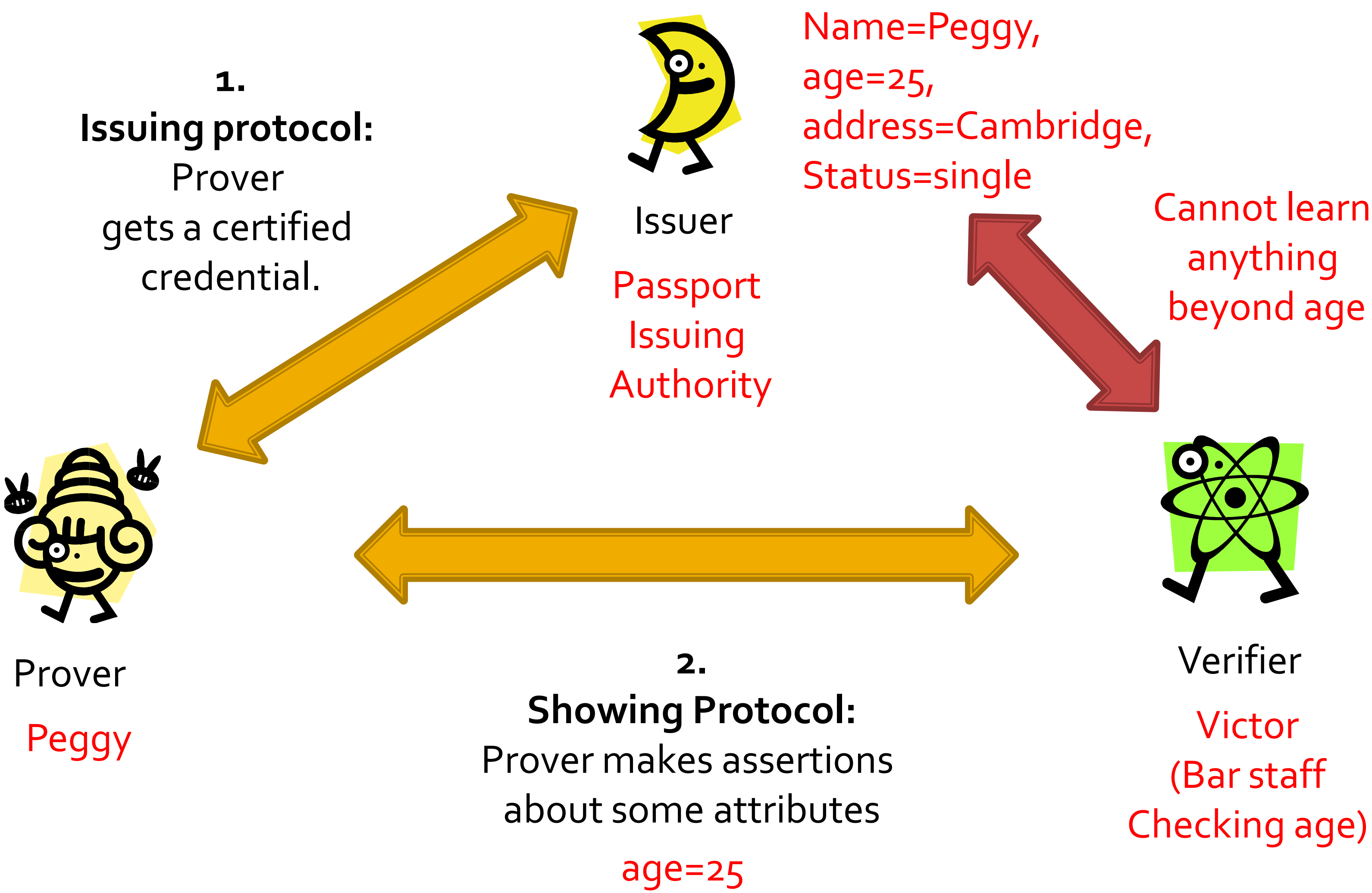
- Usual way:
  - **Identity provider** certifies attributes of a **subject**.
  - **Identity consumer** checks those attributes
  - Match credential with **live person** (biometric)

- Examples:
  - E-passport: signed attributes, with lightweight access control.
    - Attributes: nationality, names, number, pictures, …
  - Identity Cards: signatures over attributes
    - Attributes: names, date of birth, picture, address, …

Anonymous credentials

- The players:
  - Issuer (I) = Identity provider
  - Prover (P) = subject
  - Verifier (V) = identity consumer
- Properties:
  - The prover convinces the verifier that he holds a credential with attributes that satisfy some boolean formula:
    - Simple example “age=18 AND city=Cambridge”
  - Prover cannot lie
  - Verifier cannot infer anything else aside the formula
  - Anonymity maintained despite collusion of V & I

The big picture



Two flavours of credentials

- Single-show credential (Brands & Chaum)
  - Blind the issuing protocol
  - Show the credential in clear
  - Multiple shows are linkable – BAD
  - Protocols are simpler – GOOD

We will Focus on these

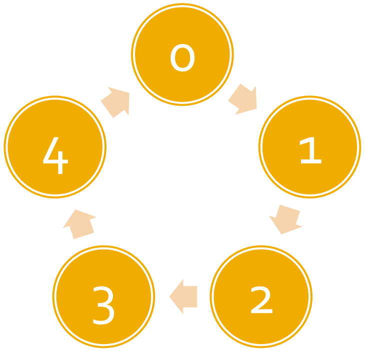
- Multi-show (Camenisch & Lysyanskaya)
  - Random oracle free signatures for issuing (CL)
  - Blinded showing
    - Prover shows that they know a signature over a particular ciphertext.
  - Cannot link multiple shows of the credential
  - More complex – no implementations

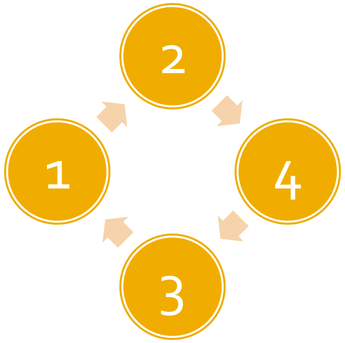
Technical Outline

- Cryptographic preliminaries
  - The discrete logarithm problem
  - Schnorr's Identification protocol
    - Unforgeability, simulator, Fiat-Shamir Heuristic
    - Generalization to representation
- Showing protocol
  - Linear relations of attributes
  - AND-connective
- Issuing protocol
  - Blinded issuing

Discrete logarithms (I) - revision

- Assume p a large prime
  - (>1024 bits—2048 bits)
  - Detail: $p = qr+1$ where q also large prime
  - Denote the field of integers modulo p as Z_p
- Example with p=5
  - Addition works fine: $1+2 = 3$, $3+3 = 1$, …
  - Multiplication too: $2*2 = 4$, $2*3 = 1$, …
  - Exponentiation is as expected: $2^2 = 4$
- Choose g in the multiplicative group of Z_p
  - Such that g is a generator
  - Example: g=2





Discrete logarithms (II) -revision

- Exponentiation is computationally easy:
  - Given g and x, easy to compute g^x
- But logarithm is computationally hard:
  - Given g and g^x, difficult to find $x = \log_g g^x$
  - If p is large it is practically impossible
- Related DH problem
  - Given (g, g^x, g^y) difficult to find g^{xy}
  - Stronger assumption than DL problem

More on Z_p

- Efficient to find inverses
  - Given c easy to calculate g^{-c} mod p
    - (p-1) – c mod p-1
- Efficient to find roots
  - Given c easy to find $g^{1/c}$ mod p
    - c (1/c) = 1 mod (p-1)
  - Note the case N=pq (RSA security)
- No need to be scared of this field.

Schnorr's Identification protocol

- Exemplary of the zero-knowledge protocols credentials are based on.
- Players
  - Public – g a generator of Z_p
  - Prover – knows x (secret key)
  - Verifier – knows $y = g^x$ (public key)
- Aim: the prover convinces the verifier that she knows an x such that $g^x = y$
  - Zero-knowledge – verifier does not learn x!
- Why identification?
  - Given a certificate containing y

Schnorr's protocol

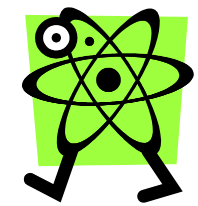
Public: g, p

Knows: x

Peggy
(Prover)

Random: w

Knows: $y=g^x$



Victor
(Verifier)

P->V: $g^w = a$ (witness)

V->P: c (challenge)

P->V: $cx+w = r$ (response)

Check:

$$g^r = y^c a$$

$$\downarrow$$

$$g^{cx+w} = (g^x)^c g^w$$

No Schnorr Forgery (intuition)

- Assume that Peggy (Prover) does not know x?
  - If, for the same witness, Peggy forges two valid responses to two of Victor's challenges

$$r_1 = c_1 x + w$$

$$r_2 = c_2 x + w$$

  - Then Peggy must know x
    - 2 equations, 2 unknowns (x,w) – can find x

Zero-knowledge (intuition)

- The verifier learns nothing new about x.
- How do we go about proving this?
  - Verifier can simulate protocol executions
    - On his own!
    - Without any help from Peggy (Prover)
  - This means that the transcript gives no information about x
- How does Victor simulate a transcript?
  - (Witness, challenge, response)

Simulator

- Need to fake a transcript ($g^{w'}$, c', r')
- Simulator:
  - Trick: do not follow the protocol order!
  - First pick the challenge c'
  - Then pick a random response r'
    - Then note that the response must satisfy:
      $g^{r'} = (g^x)^{c'} g^{w'} \rightarrow g^{w'} = g^{r'} / (g^x)^{c'}$
  - Solve for $g^{w'}$
- Proof technique for ZK
  - but also important in constructions (OR)

Non-interactive proof?

- Schnorr's protocol
  - Requires interaction between Peggy and Victor
  - Victor cannot transfer proof to convince Charlie
    - (In fact we saw he can completely fake a transcript)
- **Fiat-Shamir Heuristic**
  - $H[\cdot]$ is a cryptographic hash function
  - Peggy sets $c = H[g^w]$
  - Note that the simulator cannot work any more
    - g^w has to be set first to derive c
- Signature scheme
  - Peggy sets $c = H[g^w, M]$

Generalise to DL represenations

- Traditional Schnorr
  - For fixed g, p and public key $h = g^x$
  - Peggy proves she knows x such that $h = g^x$
- General problem
  - Fix prime p, generators $g_1, \ldots, g_l$
  - Public key $h' = g_1^{x_1} g_2^{x_2} \ldots g_l^{x_l}$
  - Peggy proves she knows $x_1, \ldots, x_l$ such that $h' = g_1^{x_1} g_2^{x_2} \ldots g_l^{x_l}$

DL represenation – protocol

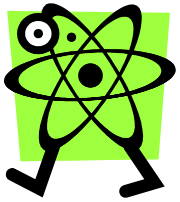
Public: g, p

Knows: $x_1, \ldots, x_l$

Peggy
(Prover)

l random: w_i

Knows:
$h = g_1^{x_1} g_2^{x_2} \ldots g_l^{x_l}$



Victor
(Verifier)

P->V: $\prod_{0<i<l} g^{w_i} = a$ (witness)

V->P: c (challenge)

$r_i = cx_i + w_i$

P->V: $r_1, \ldots, r_l$ (response)

Check:

$$\left(\prod_{0<i<l} g_i^{r_i}\right) = h^c a$$

Let's convince ourselves: $\left(\prod_{0<i<l} g_i^{r_i}\right) = \left(\prod_{0<i<l} g_i^{x_i}\right)^c \left(\prod_{0<i<l} g^{w_i}\right) = h^c a$

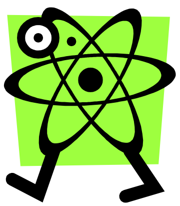
DL represenation vs. Schnorr

Public: g, p

Knows: $x_1, \ldots, x_l$

Peggy
(Prover)

Knows:
$h = g_1^{x_1} g_2^{x_2} \ldots g_l^{x_l}$



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Credentials – showing

- Relation to DL representation
- Credential representation:
  - Attributes x_i
  - Credential $h = g_1^{x_1} g_2^{x_2} \ldots g_l^{x_l}$, $Sig_{Issuer}(h)$
- Credential showing protocol
  - Peggy gives the credential to Victor
  - Peggy proves a statement on values x_i
    - $X_{age} = 28$ AND $x_{city} = H[Cambridge]$
  - Merely DL rep. proves she knows x_i

Linear relations of attributes (1)

- Remember:
  - Attributes x_i , $i = 1,\dots,4$
  - Credential $h = g_1^{x_1} g_2^{x_2} g_3^{x_3} g_4^{x_4}$, $Sig_{Issuer}(h)$
- Example relation of attributes:
  - $(x_1 + 2x_2 - 10x_3 = 13)$ AND $(x_2 - 4x_3 = 5)$
  - Implies: $(x_1 = 2x_3+3)$ AND $(x_2 = 4x_3+5)$
  - Substitute into h
    - $h = g_1^{2x_3+3} g_2^{4x_3+5} g_3^{x_3} g_4^{x_4} = (g_1^3 g_2^5)(g_1^2 g_2^4 g_3)^{x_3} g_4^{x_4}$
    - Implies: $h / (g_1^3 g_2^5) = (g_1^2 g_2^4 g_3)^{x_3} g_4^{x_4}$

Linear relations of attributes (2)

- Example (continued)
  - $(x_1 + 2x_2 - 10x_3 = 13)$ AND $(x_2 - 4x_3 = 5)$
  - Implies: $h / (g_1^3 g_2^5) = (g_1^2 g_2^4 g_3)^{x_3}\, g_4^{x_4}$
- How do we prove that in ZK?
  - DL representation proof!
    - $h' = h / (g_1^3 g_2^5)$
    - $g_1' = g_1^2 g_2^4 g_3$ $\qquad g_2' = g_4$
  - Prove that you know x_3 and x_4 such that $h' = (g_1')^{x_3} (g_2')^{x_4}$

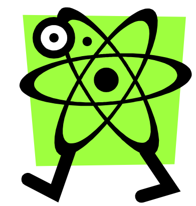
DL rep. – credential show example

Public: g, p

Knows: x_1, x_2, x_3, x_4

Peggy
(Prover)

Knows:
$h = g_1^{x_1} g_2^{x_2} g_3^{x_3} g_4^{x_4}$



Victor
(Verifier)

random: w_1, w_2

P->V: $g_1'^{w_1} g_2'^{w_2} = a'$ (witness)

V->P: c (challenge)

P->V: r_1, r_2 (response)

$r_1 = cx_3 + w_1$

$r_2 = cx_4 + w_2$

Check:

$$(g_1')^{r_1} (g_2')^{r_2} = (h')^c a$$

Check $(g_1')^{r_1} (g_2')^{r_2} = (h')^c a$

- Reminder
  - $h = g_1^{x_1} g_2^{x_2} g_3^{x_3} g_4^{x_4}$
  - $h' = h / (g_1^3 g_2^5)$ $\quad g_1' = g_1^2 g_2^4 g_3$ $\quad g_2' = g_4$
  - $a = g_1'^{w_1} g_2'^{w_2}$ $\quad r_1 = cx_3 + w_1$ $\quad r_2 = cx_4 + w_1$
- Check:
  - $(g_1')^{r_1} (g_2')^{r_2} = (h')^c a \Rightarrow$
    $(g_1')^{(\cancel{c}x_3+\cancel{w}_1)} (g_2')^{(\cancel{c}x_4+\cancel{w}_1)} = (h / (g_1^3 g_2^5))^{\cancel{c}} \cancel{g_1}'^{w_1} \cancel{g_2}'^{w_2} \Rightarrow$
    $(g_1^{2x_3+3} g_2^{4x_3+5} g_3^{x_3} g_4^{x_4}) = h$

x_1 $\quad$ x_2

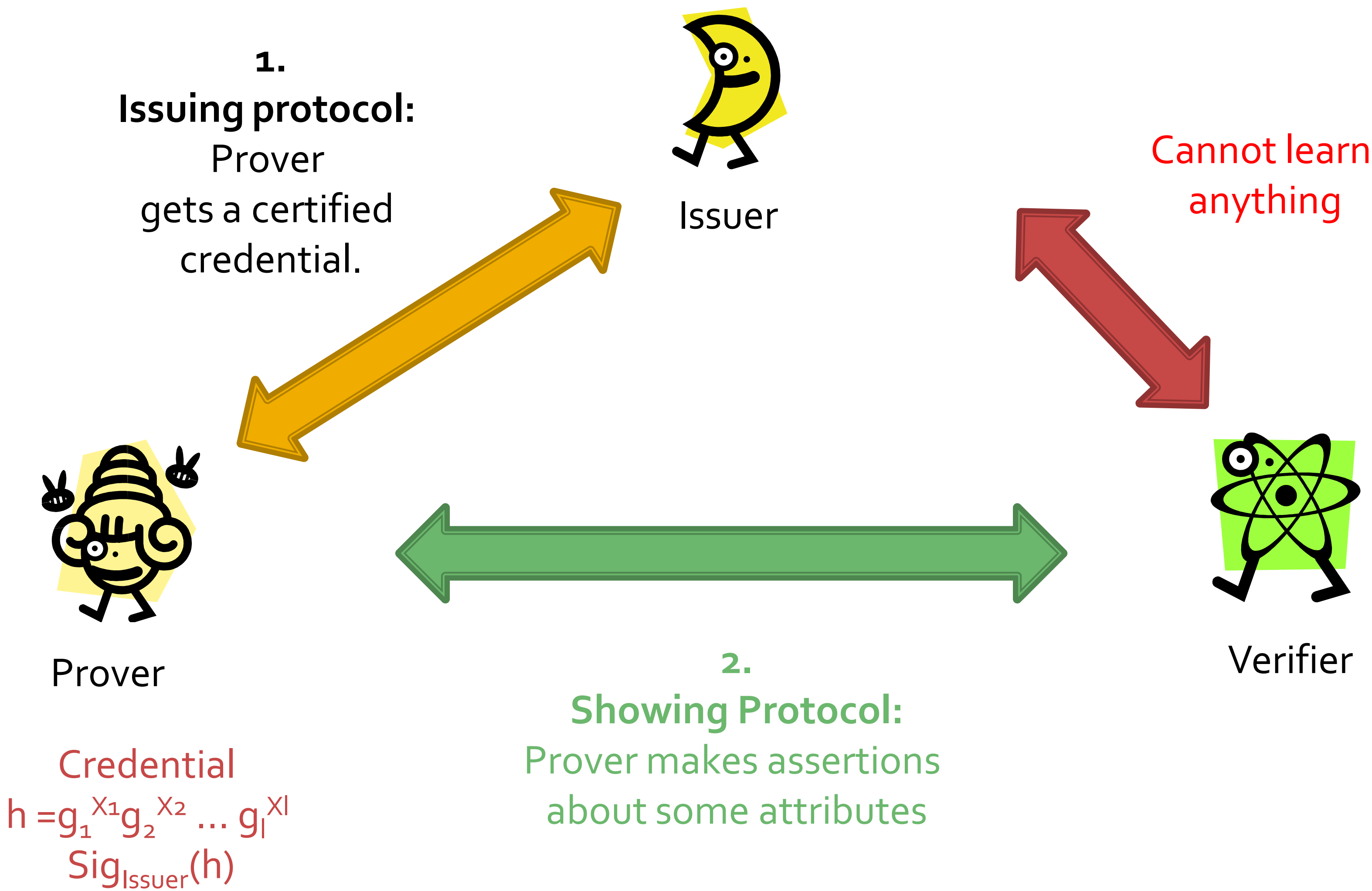
A few notes

- Showing any relation implies knowing all attributes.
- Can make non-interactive (message m)
  - c = H[h, m, a’]
- Other proofs:
  - (OR) connector *(simple concept)*
    - $(x_{age}$=18 AND x_{city}=H[Cambridge]) OR $(x_{age}$=15)
  - (NOT) connector
  - Inequality $(x_{age} > 18)$ (Yao’s millionaire protocol)

Summary of key concepts (1)

- Standard tools
  - Schnorr – ZK proof of knowledge of discrete log.
  - DL rep. – ZK proof of knowledge of representation.
- Credential showing
  - representation + certificate
  - ZK proof of linear relations on attributes (AND)
  - More reading: (OR), (NOT), Inequality

Issuing credentials



Issuing security

- Prover cannot falsify a credential
- Unlinkability
  - Issuer cannot link a showing transcript to an instance of issuing
  - h, Sig_{issuer}(h) have to be unlinkable to issuing
- Achieving unlinkability
  - Issuer's view: $h = g_1^{x_1} g_2^{x_2} \ldots g_l^{x_l}$
  - Prover uses: $h' = g_1^{x_1} g_2^{x_2} \ldots g_l^{x_l} g_0^{a_1}$

Issuing protocol – gory details

Knows: $x_1, x_2, \ldots, x_l$

Public: g, p

Knows: $x_1, x_2, \ldots, x_l$

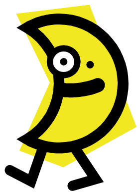
Prover

Private: $x_o, (y_1, \ldots, y_l)$

Public: $h_o = g^{x_o}, g_i = g^{y_i}$

Rand: w_o

I->P: $g^{w_o} = a_o$ (witness)



Issuer

Rand: a_1, a_2, a_3

$h' = h \cdot g^{a_1}$

$c'_o = H[h', g^{a_2}(h_o h)^{a_3} a_o]$

$c_o = c'_o + a_3$

P->I: c_o (challenge)

$r_o = c_o(x_o + \sum_i x_i y_i) + w_o$

I->P: r_o (response)

Check: $g^{r_o} = (h_o h)^{c_o} a_o$

$r'_o = r_o + a_2 + c'_o a_1$

Credential: $h' = g^{a_1} \prod g_i^{x_i}$

Signature: (c'_o, r'_o)

Check: $c'_o = H[h', g^{r'_o}(h_o h')^{-c'_o}]$

Issuing protocol – Issuer side

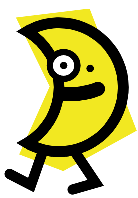
Knows: $x_1, x_2, \ldots, x_l$

Public: g, p

Private: $x_o, (y_1, \ldots, y_l)$
Public: $h_o = g^{x_o}$, $g_i = g^{y_i}$
Rand: w_o

I->P: $g^{w_o} = a_o$ (witness)

~~Non interactive signature: $c_o = H[h, a_o]$~~



Issuer

P->I: c_o (challenge)

$r_o = c_o(x_o + \sum_i x_i y_i) + w_o$

I->P: r_o (response)

ZK knowledge proof of the representation of $h_o h = g^{x_o} \prod g_i^{x_i} = g^{(x_o + \sum_i x_i y_i)}$: Just Schnorr !

Issuing protocol – Prover side (1)

Public: $g, p, h_0 = g^{x_0}, g_i = g^{y_i}$

Knows: $x_1, x_2, \ldots, x_l$

Prover

I->P: $g^{w_0} = a_0$ (witness)

P->I: c_0 (challenge)

I->P: r_0 (response)

Rand: a_1, a_2, a_3

$h' = h \cdot g^{a_1}$

$c'_0 = H[h', g^{a_2}(h_0 h)^{a_3} a_0]$

$c_0 = c'_0 + a_3$

Check: $g^{r_0} = (h_0 h)^{c_0} a_0$

$r'_0 = r_0 + a_2 + c'_0 a_1$

Schnorr Verification:

Issuer knows the representation of $(h_0 h)$!

Credential: $h' = g^{a_1} \prod g_i^{x_i}$

Signature: (c'_0, r'_0)

Check: $c'_0 = H[h', g^{r'_0}(h_0 h')^{-c'_0}]$

Issuing protocol – Prover side (2)

Public: $g, p, h_o = g^{x_o}, g_i = g^{y_i}$

Knows: $x_1, x_2, \ldots, x_l$


Prover

I->P: $g^{w_o} = a_o$ (witness)

Rand: a_1, a_2, a_3

$h' = h \cdot g^{a_1}$

$c'_o = H[h', g^{a_2}(h_o h)^{a_3} a_o]$

$c_o = c'_o + a_3$

1) Set c_o

P->I: c_o (challenge)

I->P: r_o (response)

Check: $g^{r_o} = (h_o h)^{c_o} a_o$

$r'_o = r_o + a_2 + c'_o a_1$

2) Get r_o such that…

Unlinkable

Credential: $h' = g^{a_1} \prod g_i^{x_i}$

Signature: (c'_o, r'_o)

Check: $c'_o = H[h', g^{r'_o}(h_o h')^{-c'_o}]$

My Goal

Issuing protocol – Prover side (3)

Public: $g, p, h_o = g^{x_o}, g_i = g^{y_i}$

Knows: $x_1, x_2, \ldots, x_l$

Prover

I->P: $g^{w_o} = a_o$ (witness)

Rand: a_1, a_2, a_3

$h' = h \cdot g^{a_1}$

$c'_o = H[h', g^{a_2}(h_o h)^{a_3} a_o]$

P->I: c_o (challenge)

$c_o = c'_o + a_3$

I->P: r_o (response)

Check: $g^{r_o} = (h_o h)^{c_o} a_o$

$r'_o = r_o + a_2 + c'_o a_1$

?

Credential: $h' = g^{a_1} \prod g_i^{x_i}$

Signature: (c'_o, r'_o)

Check: $c'_o = H[h', g^{r'_o}(h_o h')^{-c'_o}]$

Check

- Goal:
  - $c'_o = H[h', g^{a_2}(h_o h)^{a_3} a_o] = H[h', g^{r'_o}(h_o h')^{-c'_o}]$
    - So $g^{a_2}(h_o h)^{a_3} a_o = g^{r'_o}(h_o h')^{-c'_o}$ must be true
  - Lets follow:
    - $g^{r'_o}(h_o h')^{-c'_o} = g^{a_2}(h_o h)^{a_3} a_o \Leftrightarrow$
    - $g^{(r_o + a_2 + c'_o a_1)} (h_o h)^{-(c_o + a_3)} g^{-c_o a_1} = g^{a_2}(h_o h)^{a_3} a_o \Leftrightarrow$
    - $(g^{r_o}(h_o h)^{-c_o}) \cancel{(g^{a_2}(h_o h)^{a_3})} = \cancel{(g^{a_2}(h_o h)^{a_3})} a_o \Leftrightarrow$

Substitute r'_o and c'_o

TRUE

Unlinkability

- Issuer sees: c_o, r_o, h
  - Such that $g^{r_o} = (h_o h)^{c_o} a_o$
- Verifier sees: c'_o, r'_o, h'
- Relation:
  - Random: a_1, a_2, a_3
    - $h' = h \cdot g^{a_1}$
    - $c_o = c'_o + \mathbf{a_3}$
    - $r'_o = r_o + \mathbf{a_2} + c'_o a_1$
- Even if they collude they cannot link the credential issuing and showing

Notes on issuer

- Authentication between Issuer and Peggy
  - Need to check that Peggy has the attributes requested

- Issuing protocol should not be run in parallel!
  - (simple modifications are required)

Full credential protocol

- Putting it all together:
  - Issuer and Peggy run the issuing protocol.
    - Peggy gets:

Credential: $h' = g^{a_1} \prod g_i^{x_i}$

Signature: (c'_o, r'_o)
Check: $c'_o = H[h', g^{r'_o}(h_o h')^{-c'_o}]$

  - Peggy and Victor run the showing protocol
    - Victor checks the validity of the credential first
    - Peggy shows some relation on the attributes
      - (Using DL-rep proof on h')

Key concepts so far (2)

- Credential issuing
  - Proof of knowledge of DL-rep & x_o of issuer
  - Peggy assists & blinds proof to avoid linking
- Further topics
  - Transferability of credential
  - Double spending

Key applications

- Attribute based access control
- Federated identity management
- Electronic cash
  - (double spending)
- Privacy friendly e-identity
  - Id-cards & e-passports
- Multi-show credentials!

References

- Core:
  - Claus P. Schnorr. **Efficient signature generation by smart cards**. Journal of Cryptology, 4:161—174, 1991.
  - Stefan Brands. **Rethinking public key infrastructures and digital certificates – building in privacy**. MIT Press.
- More:
  - Jan Camenisch and Markus Stadler. **Proof systems for general statements about discrete logarithms.** Technical report TR 260, Institute for Theoretical Computer Science, ETH, Zurich, March 1997.
  - Jan Camenisch and Anna Lysianskaya. **A signature scheme with efficient proofs.** (CL signatures)

OR proofs

- Peggy wants to prove (A OR B)
  - Say A is true and B is false
- Simple modification of Schnorr
  - Peggy sends witness
  - Victor sends commitment c
  - Peggy uses <u>simulator</u> for producing a response r_B for B
    - (That sets a particular c_B)
    - Peggy chooses c_A such that $c = c_A + c_B$
  - Then she produces the response r_A for A
- Key concept: simulators are useful, not just proof tools!

Strong(er) showing privacy

- Designated verifier proof
  - A OR knowledge of verifier's key
  - Simulate the second part
  - Third parties do nor know if A is true or the statement has been built by the designated verifier!
- Non-interactive proof not transferable!